

STABILITY ANALYSIS OF SIR EPIDEMIC MODEL WITH LIMITED MEDICAL RESOURCES REVISITED

Nemat Nyamoradi¹ and Mohammad Javidi²

¹Department of Mathematics, Faculty of Sciences, Razi University, 67149 Kermanshah,
Iran

²Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran

Abstract. In this paper, the dynamics of an SIR epidemic model is explored in order to understand how the limited medical resources and their supply efficiency affect the transmission of infectious diseases. The study reveals that, with varying amount of medical resources and their supply efficiency, the target model admits backward bifurcation. Sufficient criteria are established for the existence of backward bifurcation, the existence and the stability. Numerical simulations are presented to illustrate the analytical results.

Keywords. SIR model, Stability, Basic reproduction number, Backward bifurcation.

AMS (MOS) subject classification: 34D23, 92D30.

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email: journal@monotone.uwaterloo.ca
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