

THE EXISTENCE OF GLOBAL WEAK SOLUTIONS FOR A GENERALIZED NOVIKOV EQUATION

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Abstract. A generalization of the Novikov equation is investigated. Applying the limit of viscous approximations for the equation, we establish its existence of global weak solutions in the space $C([0, \infty) \times R) \cap L^\infty([0, \infty); H^1(R))$ under certain conditions. Our analysis depends on that we derive a one-sided super bound and a higher-norm estimate on the first order derivatives of the solution.

Keywords. Global weak solution; Existence, Generalized Novikov equation.

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