

ON NONLINEAR FRACTIONAL Q -DIFFERENCE EQUATIONS INVOLVING TWO FRACTIONAL ORDERS WITH THREE-POINT NONLOCAL BOUNDARY CONDITIONS

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Abstract. We investigate the existence of solutions for a nonlinear fractional q -difference equation involving two fractional orders and both non-integral and integral nonlinearities with three-point nonlocal boundary conditions. The existence results are established by means of some classical tools of fixed point theory. An illustrative example is also discussed.

Keywords. Fractional; q -difference equations; q -integral; nonlocal boundary conditions; existence; fixed point

AMS (MOS) subject classification: 34A08, 34B10, 34B15.

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Received January 2013; revised January 2014.

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