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ON STUDY OF THE ASYMPTOTIC BEHAVIOR OF SOME RATIONAL DIFFERENCE EQUATIONS

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Abstract. In this article, we study the periodicity, the boundedness and the global stability of the positive solutions of the following nonlinear difference equation

$$x_{n+1} = ax_n + \frac{bx_{n-1} + cx_{n-2} + fx_{n-3} + rx_{n-4}}{dx_{n-1} + ex_{n-2} + gx_{n-3} + sx_{n-4}}, \qquad n = 0, 1, 2, ...$$

where the coefficients $a, b, c, d, e, f, g, r, s \in (0, \infty)$, while the initial conditions x_{-4},x_{-3},x_{-2}, x_{-1}, x_0 are arbitrary positive real numbers. Some numerical examples will be given to illustrate our results.

Keywords. Difference equations, prime period two solution, boundedness character, locally asymptotically stable, global attractor, global stability.

AMS (MOS) subject classification: 39A10, 39A11, 39A99, 34C99.

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