

ON THE HIGH ORDER LIPSCHITZ STABILITY OF INVERSE NODAL PROBLEM FOR STRING EQUATION

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Abstract. Inverse nodal problem on the string operator is the finding the density function using nodal sequence $\{z_k^{(n)}\}$. In this paper, we solve a stability problem using nodal set of eigenfunctions and show that the space of high order density functions is homeomorphic to the partition set of the space of quasinodal sequences. Basically, this method is similar to [2] and [6] which is given for Sturm-Liouville and Hill operators, respectively.

Keywords. String equation, Inverse nodal problem, Lipschitz stability.

AMS (MOS) subject classification: 34A55, 34L05, 34L20, 34D20

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