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EXISTENCE OF HOMOCLINIC ORBITS OF A CLASS OF SECOND-ORDER DIFFERENTIAL DIFFERENCE EQUATIONS

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Abstract. This paper is concerned with the existence of homoclinic orbits of the second order differential difference equations containing both advance and retardation

$$\ddot{z}(t) - K_z(t, z(t)) + f(t, z(t+\tau), z(t), z(t-\tau)) = h(t) \quad .$$

Using critical point theory we show a nontrivial homoclinic orbit is obtained as a limit of a sequence of periodic solutions of the equation.

Keywords. Homoclinic solutions; Differential difference equation; critical point theory.

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References

[1] V. Coti Zelati, I. Ekeland and E. Séré, A variational approach to homoclinic orbits in Hamil- tonian systems, *Math. Ann*, **228**, (1990) 133-160.

[2] V.Coti Zelati and P. H. Rabinowitz, Homoclinic orbits for second order Hamiltonian systems possessing superquadratic potentials, A Second Course in Elementary Differential Equations, *J. Amer. Math. Soc*, **4**, (1991) 693-727.

[3] Y. H. Ding, Existence and multiplicity results for homoclinic solutions to a class of Hamiltonian systems, *Nonlinear Anal*, **25**, (1995) 1095-1113.

[4] Y. H. Ding and M. Girardi, Periodic and homoclinic solutions to a class of Hamiltonian systems with the potentials changing sign, *J. Math. Anal. Appl*, **189**, (1995) 585-601.

[5] Y. H. Ding and L. Jeanjean, Homoclinic orbits for a nonperiodic Hamiltonian system, *J. Diff. Eqns*, **237**, (2007) 473-490.

[6] C. J. Guo, D. O'Regan and R. Agarwal, Homoclinic orbits for singular second-order neutral differential equation, *J.Math.Anal.Appl*, **366**, (2010) 550-560.

[7] C. J. Guo, D. O'Regan and R. Agarwal, Existence of subharmonic solutions and homoclinic orbits for a class of high-order differential equations, *Applicable Analysis*, **9(7)**, (2011) 1169-1183.

[8] C. J. Guo, D. O'Regan and R. Agarwal, Existence and multiplicity of homoclinic orbits of a second-order differential difference equation via variational methods, *Scientific Publications of the State University of Novi Pazarser. A: Appl. Math. Inform and Mech*, **4(1)**, (2012) 1-15.

[9] H. Hofer and K. Wysocki, First order elliptic systems and the existence of homoclinic orbits in Hamiltonian systems, *Math. Ann*, **228**, (1990) 483-503.

[10] M. Izydorek and J. Janczewska, Homoclinic solutions for a class of the second order Hamiltonian systems, *J. Diff. Eqns*, **219**, (2005) 375-389.

[11] C. F. Kreiner and J. Zimmer, Travelling wave solutions for the discrete sine-Gordon equation with nonlinear pair interaction, *Nonlinear Anal*, **70**, (2009) 3146-3158.

[12] L. S. Pontryagin, R. V. Gamkreledze and E. F. Mischenko, *The mathematical theory of optimal processes*, Interscience, New York, 1962.

[13] P. H. Rabinowitz, Homoclinic orbits for a class of Hamiltonian systems, *Proc. Roy. Soc. Edinburgh*, **114A**, (1990) 33-38.

[14] P. H. Rabinowitz, Minimax methods in critical point theory with applications to differential equations, *Amer. Math. Soc*, Providence, RI, 1986.

[15] P. H. Rabinowitz and K. Tanaka, Some results on connecting orbits for a class of Hamiltonian systems, *Math. Z*, **206**, (1991) 472-499.

[16] A. Rustichini, Functional differential equations of mixed type: The linear autonomous case, *J. Dynam. Diff. Eqns*, **1**, (1989) 121-143.

[17] A. Rustichini, Hopf bifurcation for functional differential equations of mixed type, *J. Dynam. Diff. Eqns*, **1**, (1989) 145-177.

[18] L. S. Schulman, Correlating arrows of time Some, *Phys. Rev*, **D7**, (1973) 2868-2874.

[19] L. S. Schulman, Some differential-difference equations containing both advance and retardation, *J. Math Phys*, **15**, (1974) 295-298.

[20] E. Séré, Existence of infinitely many homoclinic orbits in Hamiltonian systems, *Math. Z*, **209**, (1993) 561-590.

[21] D. Smets and M. Willem, Solitary waves with prescribed speed on infinite lattices, *J. Funct. Anal*, **149**, (1997) 266-275.

[22] A. Szulkin and W. M. Zou, Homoclinic orbits for asymptotically linear Hamiltonian systems, *J. Funct. Anal*, **2001**, (1990) 25-41.

[23] J. A. Wheeler and P. R. Feynman, Classical electrodynamics in terms of direct interparticle action, *Rev. Modern Phys*, **21**, (1949) 425-433.

[24] J. H. Wu and X. F. Zou, Asymptotic and periodic boundary value problems of of mixed FDEs and wave solutions lattice differential equations, *J. Diff. Eqns*, **21**, (1997) 315-357.

[25] X. J. Xu, Sub-harmonics of first order Hamiltonian systems and their asymptotic behaviors, *Discrete Contin. Dyn. Syst. Ser B3*, **4**, (2003) 643-654.

[26] X. J. Xu, Homoclinic orbits for first order Hamiltonian systems with convex potentials, *Advanced Nonlinear Studies*, **6**, (2006) 399-410.

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