Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **20 (2013)** 591-623

EXISTENCE RESULTS FOR A CLASS OF HYBRID SYSTEMS WITH INFINITE DELAY

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Abstract. In this paper, the existence, uniqueness, and continuation of solutions to switched systems with infinite delay and impulses is investigated. Both time-dependent and state-dependent switching are considered. The main results on existence and uniqueness are proved by adjusting classical techniques to account for impulses, infinite delay, and switches. Extended and global existence results are given for different types of switching rules. The results found are also applicable to impulsive switched systems with finite delay. An epidemic model is presented to illustrate the results.

Keywords. Existence; Uniqueness; Extended existence; Hybrid systems; Infinite delay.

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Received September 2013; revised December 2013

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