

## CONTROLLABILITY OF ABSTRACT NEUTRAL DIFFERENTIAL EQUATIONS WITH DEVIATED ARGUMENTS

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**Abstract.** In this paper, we shall study a control system represented by an abstract neutral differential equation with deviated argument in a Banach space. We used the semigroup theory of linear operators to study the complete controllability of the given system with the help of the Banach fixed point theorem. Also, we study the nonlocal system. In the last, we give an example to illustrate the applications of the abstract results.

**Keywords.** Neutral differential equations with deviated arguments, Exact controllability, Analytic semigroup, Banach fixed point theorem.

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