PERIODS OF THE SOLUTIONS OF A DIFFERENCE EQUATION IN $\mathbb{R}_+^*$ ASSOCIATED TO A SPECIAL QRT-MAP

Guy Bastien$^{1,4}$ and Marc Rogalski$^{2,3,4}$

$^1$ Institut Mathématique de Jussieu-PRG, Université Paris 6 and CNRS, France.
bastien@math.jussieu.fr
$^2$ Laboratoire Paul Painlevé, Université de Lille 1 and CNRS, France.
$^3$ Institut Mathématique de Jussieu-PRG, Université Paris 6 and CNRS, France.
marc.rogalski@upmc.fr
$^4$ Université Paris 6, 4 pl. Jussieu, 75005 Paris, France.

Abstract. We study the periods of the solutions of the difference equation in $\mathbb{R}_+^*$

$$u_{n+1}u_n = av_n + b, \quad v_{n+1}v_n = u_{n+1} + \frac{b}{u_{n+1}}, \quad a > 0, \quad b > 0,$$

(1)

associated to the special QRT-map given by the family of elliptic cubic curves $C_{a,b}(K)$
with equations

$$xy^2 + x^2 + ay + b - Kxy = 0.$$ (2)

Using Weierstrass' functions we calculate the rotation number of the restriction of the
associated QRT-map to the positive part of a cubic $C_{a,b}(K)$ of the family (2). Using the
prime number theorem, we prove that every integer $n \geq 1$, excepted 2, 3, 4, 6, 10, is the
minimal period of some solution of (1) for some $a > 0$ and $b > 0$. The exceptional numbers
are minimal periods of no solution of (1), whatever positive values have the parameters $a$
and $b$.

Keywords. Difference equations; elliptic curves; QRT maps; rotation number; periodic
orbits.

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References


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email: journal@monotone.uwaterloo.ca
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