

## EXISTENCE OF A NONTRIVIAL SOLUTION FOR A CLASS OF NONLINEAR ELLIPTIC SYSTEMS IN $\mathbb{R}^N$

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**Abstract.** Using a variational approach, we investigate the existence of a nontrivial weak solution for a class of general capillarity systems in  $\mathbb{R}^N$ . The proofs rely essentially on the mountain pass theorem with a weak version of the Palais-Smale conditions, due to Cerami.

**Keywords.** Weak solutions; existence; capillarity systems; variational methods.

**AMS (MOS) subject classification:** 35B30, 35J60, 35P15

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