

**EXISTENCE AND MULTIPLICITY OF SOLUTIONS  
TO STURM-LIOUVILLE BOUNDARY VALUE  
PROBLEM FOR IMPULSIVE DIFFERENTIAL  
INCLUSIONS**

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**Abstract.** In this paper we consider a class of  $p$ -Laplacian Sturm-Liouville boundary value problem. Using a variational method based on nonsmooth critical point theory, we prove the existence and multiplicity of solutions.

**Keywords.** Nonsmooth critical point; Variational methods; Locally Lipschitz; Impulsive; Inclusion.

**AMS (MOS) subject classification:** 49J52, 47J30, 34A37.

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