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# DYNAMIC ANALYSIS OF A FRACTIONAL ORDER ONE–COUNTRY GAME MODEL

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**Abstract.** In this article, a new fractional order one–country game model (OCGM) with distinct incidence is explored. We propose a mathematical system to model this situation. All the feasible equilibria of the system are obtained and the conditions for the existence of the interior equilibrium are determined. The basic reproduction numbers $R_0 = \frac{\beta}{(\alpha+1)(\gamma+\mu)}$ and $R_0^* = \frac{E}{r_v}$ are found. It is shown that the disease-free, pure nonvaccinator equilibrium and disease-free, pure vaccinator equilibrium are locally asymptotically stable. It is shown that if $R_0 < 1$ and $R_0^* < 1$ then the system has an endemic, pure nonvaccinator equilibrium which is locally asymptotically. We proved that the disease-free, pure vaccinator equilibrium point is asymptotically stable if $R_0^* > 1$ hold. The endemic, pure nonvaccinator equilibrium is asymptotically stable if $R_0 < R_{01}$ hold. Also stability analysis of the system is studied by using the fractional Routh–Hurwitz stability conditions. Numerical simulations are provided to illustrate analytical results.

**Keywords.** Game model, fractional differential equation, stability, dynamic analysis, equilibrium point.

**AMS (MOS) subject classification:** 26A33

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