Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **20 (2013)** 641-656
 http://www.watam.org

ON WEAKLY NONLINEAR BOUNDARY VALUE PROBLEMS WITH IMPULSES

Daniel Maroncelli [1] and Jesús Rodríguez [2]

[1]Department of Mathematics
North Carolina State University, Raleigh, NC 27695-8205, U.S.A

[2]Department of Mathematics
North Carolina State University, Raleigh, NC 27695-8205, U.S.A

Corresponding author email:rodrigu@math.ncsu.edu

Abstract. In this paper we discuss the existence of solutions to weakly nonlinear boundary value problems of the form

$$x'(t) = A(t)x(t) + g(t) + \varepsilon f(t, x(t)), \quad t \in [0, 1] \setminus \{t_1, t_2, \cdots, t_k\}$$
$$x(t_i^+) - x(t_i^-) = w_i, \quad i = 1, ..., k$$

subject to boundary conditions

$$Bx(0) + Dx(1) = 0.$$

We present a qualitative analysis of the dependence of solutions on the "small" parameter ε. Emphasis will be placed on the resonant case.

Keywords. Boundary value problems, Contraction mapping theorem, Implicit function theorem, Lyapunov-Schmidt, Impulsive differential equations

1 Introduction

In the following we will be analyzing problems of the form

$$x'(t) = A(t)x(t) + g(t) + \varepsilon f(t, x(t)), \quad t \in [0, 1] \setminus \{t_1, t_2, \cdots, t_k\} \tag{1}$$

$$x(t_i^+) - x(t_i^-) = w_i, \quad i = 1, ..., k \tag{2}$$

subject to boundary conditions

$$Bx(0) + Dx(1) = 0. \tag{3}$$

References

[1] S. Bancroft, J.K. Hale, and D. Sweet, *Alternative problems for nonlinear functional equations*, J. Differ. Equ. **4** (1968), 40–56.

[2] A. Boucherif, *Nonlinear multipoint boundary value problems*, Nonlinear Anal. **10, No. 9** (1986), 957–964.

[3] S. Chow and J. K. Hale, *Methods of bifurcation theory*, Springer, Berlin, 1982.

[4] R. E. Gaines and J. L. Mawhin, *Coincidence degree and nonlinear differential equations (lecture notes in mathematics)*, Springer-Verlag, 1977.

[5] C. Gerhardt, *Analysis ii*, International Press, Somerville, MA, 2006.

[6] J. K. Hale, *Ordinary differential equations*, Robert E. Kreiger Publishing Company, Malabar FL, 1980.

[7] I. V. Lakshmikantham, D. D. Bainov, and P. S. Simeonov, *Theory of impulsive differential equations*, World Scientific, Singapore, 1989.

[8] Serge Lang, *Real analysis, second ed.*, Addison-Wesley Publishing Company, Reading, MA, 1983.

[9] D. C. Lewis, *On the role of first integrals in the perturbation of periodic solutions*, Ann. Math. **63** (1956), 535–548.

[10] B. Liu and J.S. Yu, *Solvability of multi-point boundary value problem at resonance (iii)*, Appl. Math. Comput. **129** (2002), 119–143.

[11] D. Maroncelli and J. Rodríguez, *On the solvability of nonlinear impulsive boundary value problems*, submitted (2012).

[12] Jesús Rodríguez, *An alternative method for boundary vallue problems with large nonlinearities*, J. Differ. Equ. **43** (1982), 157–167.

[13] Jesús Rodríguez and D. Sweet, *Projection methods for nonlinear boundary value problems*, J. Differ. Equ. **58** (1985), 282–293.

[14] Jesús Rodríguez and Padraic Taylor, *Multipoint boundary value problems for nonlinear ordinary differential equations*, Nonlinear Anal. **68** (2008), 3465–3474.

[15] A. M. Samoilenko and N. A. Perestyuk, *Impulsive differential equations*, World Scientific, Singapore, 1995.

[16] Wayne Spealman and Daniel Sweet, *The alternative method for solutions in the kernel of a bounded linear functional*, J. Differ Equ. **37** (1980), 297–302.

Received May 2012; revised October 2013.

email: journal@monotone.uwaterloo.ca
http://monotone.uwaterloo.ca/~journal/