

RESONANT EQUATIONS DRIVEN BY THE p -LAPLACIAN PLUS AN INDEFINITE POTENTIAL

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Abstract. We consider a nonlinear elliptic problem driven by the p -Laplacian plus an indefinite potential and a parametric reaction with a “concave” term, which is asymptotically resonant with respect to any eigenvalue of the differential operator. Using a combination of variational methods based on the critical point theory with Morse theory, we show that for all small values of the parameter the problem has at least three nontrivial smooth solutions.

Keywords. Resonant problem, p -Laplacian plus an indefinite potential, eigenvalues and eigenfunctions, unique continuation property, critical groups, multiplicity theorem.

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