POSITIVE SOLUTIONS FOR A SECOND ORDER IMPULSIVE BVP WITH BOUNDED LINEAR OPERATOR CONDITIONS

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Abstract. In this paper, the authors are concerned about the existence of at least one positive solution of a second order impulsive boundary value problem with bounded linear functional conditions. A recent functional-operator fixed point theorem, which is a generalization of Leggett and Williams fixed point theorem, is applied.

Keywords. Impulse; boundary value problem; positive solutions; bounded linear functional conditions; functional-operator fixed point theorem.

AMS (MOS) subject classification: 34A37, 34B18, 34B37

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