EXISTENCE AND MULTIPLICITY RESULTS OF POSITIVE SOLUTIONS FOR BOUNDARY VALUE PROBLEMS OF NONLINEAR FRACTIONAL DIFFERENTIAL EQUATION

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Abstract. This paper concerns about existence and multiplicity results of the following fractional order boundary value problem:

$$\begin{cases} {}^{c}D_0^{\alpha}u(t) = \lambda h(t)f(t, u(t)), & t \in (0,1) \ , \ \alpha \in (2,3), \\[2mm] u(0) + u^{'}(0) = 0 = u(1) + u^{'}(1), \\ au^{''}(0) + bu^{''}(1) = 0, & a > 0 \ , b \le 0 \ , a + b > 0, \end{cases}$$

where ${}^{c}D_0^{\alpha}$ represents the fractional Caputo derivative of order α. The main results involve some well known nonlinear analysis techniques to prove our claimed results. In order to illustrate the main results, at the end of each technique, an example is represented.

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