

NONLOCAL CONTROLLABILITY OF MIXED VOLTERRA-FREDHOLM TYPE FRACTIONAL SEMILINEAR INTEGRO-DIFFERENTIAL INCLUSIONS IN BANACH SPACES

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Abstract. In this paper, we establish a sufficient conditions for the nonlocal controllability of mixed Volterra- Fredholm type fractional semilinear integro-differential inclusions in Banach spaces. The results are obtained by using fractional calculus, operator semigroups and Bohnenblust-Karlin's fixed point theorem. Finally, an example is given to illustrate the theoretical results.

Keywords. Controllability, Mixed Volterra-Fredholm type fractional differential inclusions, Integro-differential equations, Nonlocal conditions, Bohnenblust-Karlin's fixed point theorem.

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