

## MULTIPLICITY OF ALMOST PERIODIC OSCILLATIONS IN A HARVESTING MUTUALISM MODEL WITH TIME DELAYS

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**Abstract.** By means of Mawhin's continuation theorem of coincidence degree theory, some new sufficient conditions are obtained for the existence of at least four positive almost periodic solutions for a harvesting mutualism model with time delays. To the best of the author's knowledge, so far, the result of this paper is completely new. An example is employed to illustrate the result of this paper.

**Keywords.** Multiplicity; Positive almost periodic solutions; Coincidence degree; Mutualism; Harvesting.

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