

THE GLOBAL STRONG SOLUTION FOR A NONLINEAR PSEUDO-PARABOLIC EQUATION

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Abstract. The initial value problem for a nonlinear generalized pseudo-parabolic equation is investigated. Using the fixed point theorem, we establish the existence and uniqueness of local solutions to the problem in the Sobolev space $C([0, T]; H^s(R)) \cap C^1([0, T]; H^{s-1}(R))$ with $s > \frac{3}{2}$. The existence of local weak solutions is proved in the space $H^s(R)$ with $1 \leq s \leq \frac{3}{2}$. Certain assumptions imposing on the initial value and the nonlinear terms in the equation are provided to guarantee the well-posedness of the global strong solution to the problem in a Sobolev space.

Keywords. Local strong solutions; Local weak solutions; Global strong solution; Pseudo-parabolic equation.

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