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OSCILLATION, NON-OSCILLATION, AND ASYMPTOTIC BEHAVIOR FOR THIRD ORDER NONLINEAR DIFFERENCE EQUATIONS

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Abstract. In this paper the dynamics for a third-order rational difference equation in the form

$$x_{n+1} = \frac{x_{n-2}x_n + x_{n-2}^2 + a}{x_{n-2}^2 x_n + x_{n-2} + a}, \quad n = 0, 1, 2, \cdots,$$

where $a \in [0, \infty)$ and the initial values $x_{-2}, x_{-1}, x_0 \in (0, \infty)$, is considered. The rule for the trajectory structure of solutions of this equation is clearly described out. The successive lengths of positive and negative semicycles of nontrivial solutions of this equation are found to occur periodically with prime period 7 and in a period with the rule represented by $\{2^-, 3^+, 1^-, 1^+\}$. By utilizing the rule, the positive equilibrium point of the equation is verified to be globally asymptotically stable.

Keywords. rational difference equation, semicycle, cycle length, global asymptotic stability.

AMS (MOS) subject classification: 39A10, 40A05.

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