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MULTIPLE POSITIVE SOLUTIONS OF SEMILINEAR ELLIPTIC SYSTEMS WITH NONLINEAR BOUNDARY CONDITIONS

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Abstract. We consider the semilinear elliptic system

$$\begin{cases} -\Delta u + m_1(x)u = f_u(x,u,v) & x \in \Omega, \\ -\Delta v + m_2(x)v = f_v(x,u,v) & x \in \Omega, \end{cases}$$

with the boundary conditions $\frac{\partial u}{\partial n} = \lambda g(x,u)$ and $\frac{\partial v}{\partial n} = \mu h(x,v)$, where $\Omega \subset \mathbb{R}^N$ is a bounded smooth domain, $\lambda, \mu > 0$ and the functions f, g, h, m_1 and m_2 satisfy some suitable conditions. Using the fibering map and by extracting the Palais-Smale sequences in the Nehari manifold, we prove that the above system has at least two distinct positive solutions when the pair (λ, μ) belongs to a certain subset of $\mathbb{R}^2$.

Keywords. critical points, nonlinear boundary value problems, semilinear elliptic system, fibering map, Nehari manifold.

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