

AN EXISTENCE AND UNIQUENESS THEOREM TO THE CAUCHY PROBLEM FOR GENERALISED SET DIFFERENTIAL EQUATIONS

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Abstract. In this paper the concept of generalized differentiability for set-valued mappings proposed by A.V. Plotnikov, N.V. Skripnik is used. The generalized set-valued differential equations with generalized derivative are considered, the existence and uniqueness theorem is proved.

Keywords. Set-valued mapping, Hukuhara derivative, generalized derivative, set differential equation, existence and uniqueness theorems.

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1 References

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