

## OSCILLATION OF THIRD-ORDER NEUTRAL DYNAMIC EQUATIONS ON TIME SCALES

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**Abstract.** In this paper, we establish some new sufficient conditions which guarantee that the solution  $x$  of a third-order neutral delay dynamic equation

$$\left( a(t) \left[ \left( r(t)(x(t) + p(t)x(g(t)))^\Delta \right)^\Delta \right]^\gamma \right)^\Delta + q(t)x^\gamma(\tau(t)) = 0$$

is either oscillatory or satisfies  $\lim_{t \rightarrow \infty} x(t) = 0$ ; here  $\gamma > 0$  is a quotient of odd positive integers,  $a$ ,  $r$ ,  $p$ , and  $q$  are real-valued positive rd-continuous functions defined on  $\mathbb{T}$  which is unbounded above. Our results, as a special case when  $\mathbb{T} = \mathbb{R}$ , include some of the results in [Math. Comput. Modelling, 52 (2010) 215–226], and improve some results obtained for third-order dynamic equations without a neutral term on time scales. In particular, the Hille-Nehari type criteria obtained not only extend some related results reported in [J. Math. Anal. Appl., 329 (2007) 112–131] but also improve those by [Sci. China Math., 54 (2011) 2597–2614] and [J. Comput. Appl. Math., 236 (2012) 2354–2366] in the case where  $\gamma \geq 1$ .

**Keywords.** Oscillatory solution; Neutral delay dynamic equation; Third-order equation; Time scale.

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