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NUMERICAL CHAOTIC BEHAVIOR OF FRACTIONAL ORDER NEWTON-LEIPNIK SYSTEM

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Abstract. The fractional order Newton–Leipnik (FONL) system can be written as $\frac{d^\alpha x}{dt^\alpha} = -ax+y+10zy$, $\frac{d^\alpha y}{dt^\alpha} = -x-0.4y+5xz$, $\frac{d^\alpha z}{dt^\alpha} = bz-5xy$ with a and b being real parameters. In this paper, stability analysis of the FONL system is studied by using the fractional Routh–Hurwitz stability conditions. We have studied the local stability of the equilibrium points of FONL system. We applied an efficient numerical method based on converting the fractional derivative to integer derivative to solve the FONL system. The chaotic behavior of the system discussed also.

Keywords. Newton–Leipnik system, Routh–Hurwitz conditions, chaos.

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