STRONG CONVERGENCE THEOREMS FOR VARIATIONAL INEQUALITY PROBLEMS AND SYSTEM OF GENERALIZED MIXED EQUILIBRIUM PROBLEMS

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Abstract. In this paper, we construct a new iterative scheme by hybrid method for approximation of common element of set of solutions of a variational inequality problem and set of common solutions to a system of generalized mixed equilibrium problems in a 2-uniformly convex real Banach space which is also uniformly smooth. Then, we prove strong convergence of the scheme to a common element of the two sets. We give several applications of our results in a Banach space. Our results extend many important recent results in the literature.

Keywords. generalized mixed equilibrium problems; variational inequality problem; hybrid method; strong convergence; Banach spaces.

AMS (MOS) subject classification: 47H06, 47H09, 47J05, 47J25.

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