

## PERIODICITY IN A DELAYED VERSION OF THE KALDOR TRADE CYCLE MODEL

M.V.S. Frasson<sup>1</sup>, M.C. Gadotti<sup>2</sup>, S.H.J. Nicola<sup>3</sup>, and P.Z. Táboas<sup>4</sup>

<sup>1</sup> Departamento de Matemática Aplicada e Estatística, ICMC-Universidade de São Paulo, Avenida Trabalhador São-carlense 400, 13566-590 So Carlos SP, BRAZIL  
e-mail: frasson@icmc.usp.br. Supported by CNPq - Processo 479747/2008-3

<sup>2</sup> Departamento de Matemática, IGCE - Universidade Estadual Paulista, Avenida 24A 1515, 13506-700 Rio Claro SP, BRAZIL  
e-mail: martacg@rc.unesp.br. Supported by FAPESP - Processo 2008/04718-5

<sup>3</sup> Departamento de Matemática, Universidade Federal de São Carlos, Rodovia Washington Luis, km 235 Norte, 13565-905 São Carlos SP, BRAZIL  
e-mail: selmaj@dm.ufscar.br

<sup>4</sup> Departamento de Matemática Aplicada e Estatística, ICMC-Universidade de São Paulo, Avenida Trabalhador São-carlense 400, 13566-590 So Carlos SP, BRAZIL  
e-mail: pztaboas@icmc.usp.br

**Abstract.** We are concerned with the Kaldor's trade cycle model under the effect of a delay which represents a gestation lag between a decision of investment and its effect on the capital stock. Taking the adjustment coefficient in the goods market as a bifurcation parameter, we achieve global branches of periodic solutions. In our setting the delay is a constant inherent to the specific economy.

**Keywords.** Delay equations; Kaldor trade model; periodic orbits; bifurcation; global branch.

**AMS (MOS) subject classification:** 39B82.

## References

- [1] M. Baptistini and P. Táboas, *On the stability of some exponential polynomials*, J. Math. Anal. Appl. **205** (1997), no. 1, 259–272. MR 1426993 (97k:34073)
- [2] M. Z. Baptistini and P. Z. Táboas, *On the existence and global bifurcation of periodic solutions to planar differential delay equations*, J. Differential Equations **127** (1996), no. 2, 391–425. MR 1389402 (97c:34137)
- [3] W. W. Chang and D. J. Smyth, *The existence and persistence of cycles in a non-linear model: Kaldor's 1940 model re-examined*, The Review of Economic Studies **38** (1971), no. 1, 37–44.
- [4] S.N. Chow and J.K. Hale, *Periodic solutions of autonomous equations*, J. Math. Anal. Appl. **66** (1978), no. 3, 495–506. MR 517743 (80a:34102)
- [5] M. V. S. Frasson and S. M. Verduyn Lunel, *Large time behaviour of linear functional differential equations*, Integral Equations Operator Theory **47** (2003), no. 1, 91–121. MR 2015849 (2004j:34141)
- [6] J. Grasman and J. J. Wentzel, *Co-existence of a limit cycle and an equilibrium in kaldor's business cycle model and its consequences*, Journal of Economic Behavior and Organization **24** (1994), no. 3, 369–377.
- [7] J. K. Hale and S.M. Verduyn Lunel, *Introduction to functional-differential equations*, Applied Mathematical Sciences, vol. 99, Springer-Verlag, New York, 1993. MR 1243878 (94m:34169)
- [8] A. Kaddar and H. Talibi Alaoui, *Hopf bifurcation analysis in a delayed Kaldor-Kalecki model of business cycle*, Nonlinear Anal. Model. Control **13** (2008), no. 4, 439–449. MR 2477046 (2009m:91131)
- [9] ———, *Global existence of periodic solutions in a delayed Kaldor-Kalecki model*, Nonlinear Anal. Model. Control **14** (2009), 463–472.
- [10] N. Kaldor, *A model of the trade cycle*, Economic Journal **50** (1940), no. 197, 78–92.
- [11] M. Kalecki, *A macrodynamic theory of business cycles*, Econometrica (1935), no. 3, 327–344.
- [12] A. Krawiec and M. Szydłowski, *The Kaldor-Kalecki business cycle model*, Ann. Oper. Res. **89** (1999), 89–100, Nonlinear dynamical systems and adaptive methods (Vienna, 1997). MR 1704047
- [13] Y. Kuang, *Delay differential equations with applications in population dynamics*, Academic Press, London, 1993.
- [14] R.D. Nussbaum, *A global bifurcation theorem with applications to functional differential equations*, J. Functional Analysis **19** (1975), no. 4, 319–338. MR 0385656 (52 #6516)
- [15] S. Ruan and J. Wei, *Periodic solutions of planar systems with two delays*, Proc. Roy. Soc. Edinburgh Sect. A **129** (1999), 1017–1032.
- [16] M. Szydłowski and A. Krawiec, *The Kaldor-Kalecki model of business cycle as a two-dimensional dynamical system*, J. Nonlinear Math. Phys. **8** (2001), no. suppl., 266–271, Nonlinear evolution equations and dynamical systems (Kolimbar, 1999). MR 1821542
- [17] ———, *The stability problem in the Kaldor-Kalecki business cycle model*, Chaos Solitons Fractals **25** (2005), no. 2, 299–305. MR 2131320 (2005k:91224)
- [18] P. Táboas, *Periodic solutions of a planar delay equation*, Proc. Roy. Soc. Edinb. A (1990), no. 116, 85–101.
- [19] Y. Takeuchi and T. Yamamura, *Stability analysis of the Kaldor model with time delays: monetary policy and government budget constraint*, Nonlinear Anal. Real World Appl. **5** (2004), no. 2, 277–308. MR 2025069 (2005e:91128)

Received October 2012; revised May 2013.

<http://monotone.uwaterloo.ca/~journal/>