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SIGN-CHANGING SOLUTIONS OF SECOND ORDER DIRICHLET PROBLEM WITH IMPULSE EFFECTS

Ruyun Ma[1] and Jinyi Sun[2] and Mohammed Elsanosi[3]

[1,2,3]Department of Mathematics, Northwest Normal University,
Lanzhou 730070, P R China

Corresponding author email:mary@nwnu.edu.cn

Abstract. We consider the second order boundary value problems with impulse effects

$$\begin{cases} u''(t)+\lambda a(t)f(u(t))=0, \quad t\in(0,1)\backslash\{t_1,\ t_2,\cdots,\ t_m\}, \\ \triangle u(t_k)=\alpha_k u(t_k), \quad k=1,2,\cdots,m, \\ u(0)=u(1)=0, \end{cases}$$

where $\lambda>0$ is a parameter, $\alpha_k>-1$, and $0<t_1<t_2<\cdots<t_m<1$ are given constants, $\Delta u(t_k)=u(t_k^+)-u(t_k^-)$, $u(t_k^+)$ (respectively $u(t_k^-)$) denote the right limit (respectively left limit) of $u(t)$ at $t=t_k$; $a:[0,1]\to[0,\infty)$ is continuous and $a(\cdot)\not\equiv 0$ on any subinterval of [0,1]; $f:R\to R$ is continuous, and there exist two constants $s_2<0<s_1$ such that $f(s)=0,\ s\in[\beta s_2,\alpha s_2]\cup\{0\}\cup[\alpha s_1,\beta s_1]$ and $f(s)s>0$ for $s\in R\backslash\ \{[\beta s_2,\alpha s_2]\cup\{0\}\cup[\alpha s_1,\beta s_1]\}$ for some constants α and β, the limits $f_0=\lim\limits_{|s|\to 0}\frac{f(s)}{s}, f_\infty=\lim\limits_{|s|\to\infty}\frac{f(s)}{s}$ exist. We show the existence of sign-changing solutions via global bifurcation techniques.

Keywords. Dirichlet problem; sign-changing solution; multiplicity; impulsive effects; bifurcation.

AMS (MOS) subject classification: 34B37

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