Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **20 (2013)** 197-226
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DOUBLE ZERO BIFURCATION WITH HUYGENS SYMMETRY

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Abstract. This paper presents a study of the effects of symmetry on the generic bifurcation at a double-zero eigenvalue that was first investigated by Bogdanov and Takens. Two different symmetry groups are considered: Huygens symmetry and odd-Huygens symmetry. Here Huygens symmetry means that the system is equivariant under permutation of the two state variables. Using Hilbert-Weyl theory, normal forms are given for each symmetry group. The normal forms are further simplified using Gavrilov's transformation, and formulae are presented that allow identification of the normal form parameters in terms of the coefficients of the original system. Complete sets of codimension-two bifurcation diagrams with representative phase portraits are presented, for both symmetries. These diagrams exhibit codimension-one bifurcations including saddlenode, pitchfork, Hopf and heteroclinic. The effects of symmetry-breaking perturbations on these codimension-one bifurcations are analyzed. The results presented here contrast strongly with the classical results of Bogdanov and Takens.

Keywords. Double zero bifurcation; Bogdanov-Takens bifurcation; equivariant bifurcation theory; Huygens symmetry; Hilbert basis theorem.

AMS (MOS) subject classification: 34C23; 37G05; 37G40.

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Received July 2012; revised January 2013.

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