

EXISTENCE AND MULTIPLICITY OF NONTRIVIAL NONNEGATIVE SOLUTIONS FOR A CLASS OF QUASILINEAR P -LAPLACIAN SYSTEMS

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Abstract. We consider the existence and multiplicity of nonnegative solutions for the following nonhomogeneous elliptic system

$$\begin{cases} -\Delta_p u + m_1(x)|u|^{p-2}u = \frac{1}{q}f_u(x, u, v) + g_u(x, u, v) & x \in \Omega, \\ -\Delta_p v + m_2(x)|v|^{p-2}v = \frac{1}{q}f_v(x, u, v) + g_v(x, u, v) & x \in \Omega, \end{cases}$$

with boundary conditions $|\nabla u|^{p-2} \frac{\partial u}{\partial n} = \frac{1}{r} \lambda h_u(x, u, v)$ and $|\nabla v|^{p-2} \frac{\partial v}{\partial n} = \frac{1}{r} \lambda h_v(x, u, v)$, where $\Omega \subset \mathbb{R}^N$ is a bounded domain in \mathbb{R}^N with smooth boundary $\partial\Omega$, Δ_p denotes the p -Laplacian operator defined by $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$, $1 \leq q < p < r < \frac{pN}{N-p}$, $\lambda > 0$ and f , g and h are positively homogeneous C^1 -functions of degrees q , 1 and r , respectively. By using the fibering maps and the Nehari manifold associated with the Euler functional for the problem, we prove that there exists λ^* such that for $\lambda \in (0, \lambda^*)$, the above problem has at least two nontrivial nonnegative solutions.

Keywords. critical points, nonlinear boundary value problems, quasilinear p -Laplacian system, fibering map, Nehari manifold.

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