

THE GLOBAL WELL-POSEDNESS OF WEAK SOLUTIONS FOR THE DEGASPERIS-PROCESI EQUATION WITHOUT PEAKONS

Shaoyong Lai and Aiyin Wang

Department of Mathematics

Southwestern University of Finance and Economics, Chengdu, 610074, China

Abstract. The Degasperis-Procesi equation with no peakon solutions is investigated. Conditions on the coefficient k of the term u_x in the equation and the initial value u_0 are derived to guarantee the existence and uniqueness of global weak solutions in the lower order Sobolev space $H^s(R)$ with $1 \leq s \leq \frac{3}{2}$. In addition, provided $|u_0|$ and $|u_{0xx}|$ are bounded, it is shown that the Degasperis-Procesi equation always has a unique global weak solution in $H^s(R)$ with $1 \leq s \leq \frac{3}{2}$ if the positive constant k is suitable large.

Keywords. Degasperis-Procesi equation; No peakons; Global weak solutions.

AMS (MOS) subject classification: 35G25;35L05

References

- [1] A Degasperis, D. Holm and A. Hone, A new integral equation with peakon solutions, *Theor. Math. Phys.*, **133**, (2002) 1461-1472.
- [2] R. Camassa and D. Holm, An integrable shallow water equation with peaked solitons, *Phys. Rev. Lett.*, **71**, (1993) 1661-1664.
- [3] H. R. Dullin, G. A. Gottwald and D. D. Holm, Korteweg-de Vries and other asymptotically equivalent equations for shallow water waves, *Fluid Dynamic Research*, **33**, (2003) 73-79.
- [4] A. Constantin and D. Lannes, The hydro-dynamical relevance of the Camassa-Holm and Degasperis-Procesi equations, *Arch. Ration. Mech. Anal.*, **193**, (2009) 165-186.
- [5] H. Lundmark and J. Szmigielski, Multi-peakon solutions of the Degasperis-Procesi equation, *Inverse Problems*, **19**, (2003) 1241-1245.
- [6] V.O. Vakhnenko and E. J. Parkes, Periodic and solitary-wave solutions of the Degasperis-Procesi equation, *Chaos Solitons Fractals*, **20**, (2004) 1059-1073.
- [7] D. D. Holm and M. F. Staley, Wave structure and nonlinear balances in a family of evolutionary PDEs, *SIAM J. Appl. Dyn. Syst.*, **2**, (2003) 323-380.
- [8] Z. W. Lin and Y. Liu, Stability of peakons for the Degasperis-Procesi equation, *Comm. Pure Appl. Math.*, **62**, (2009) 125-146.
- [9] Z. Y. Yin, On the Cauchy problem for an integrable equation with peakon solutions, *Illinois J. Math.*, **47**, (2003) 649-666.
- [10] Z. Y. Yin, Global weak solutions for a new periodic integrable equation with peakon solutions, *J. Func. Anal.*, **212**, (2004) 182-194.
- [11] L. Lenells, Traveling wave solutions of the Degasperis-Procesi equation, *J. Math. Anal. Appl.*, **306**, (2005) 72-82.
- [12] Y. Matsuno, Multisoliton solutions of the Degasperis-Procesi equation and their peakon limit, *Inverse Problems*, **21**, (2005) 1553-1570.
- [13] G. M. Coclite and K. H. Karlsen, On the well-posedness of the Degasperis-Procesi equation, *J. Funct. Anal.*, **223**, (2006) 60-91.
- [14] F. Guo, B. F. Feng, H. J. Gao and Y. Liu, On the initial value problem to the Degasperis-Procesi equation with linear dispersion, *Discrete and Continuous Dynamical Systems*, **26**, (2010) 1269-1290.
- [15] S. Y. Lai and Y. W. Wu, The local well-posedness and existence of weak solutions for a generalized Camassa-Holm equation, *J. Differential Equations*, **248**, (2010) 2038-2063.
- [16] T. Kato and G. Ponce, Commutator estimates and the Euler and Navier-Stokes equations, *Comm. Pure Appl. Math.*, **41**, (1998) 891-907.
- [17] C.B. Ruscitti, Existence and uniqueness for viscous Boussinesq system in abstract Banach spaces, *Dynamics of Continuous, Discrete and Impulsive systems-A*, **17**, (2010) 659-675.
- [18] I. Altun and D. Turkoglu, An existence theorem for the common solutions for a pair of integral inclusions, *Dynamics of Continuous, Discrete and Impulsive systems-A*, **18**, (2011) 135-147.
- [19] S. Migorski, Existence of solutions to nonlinear second order evolution inclusions without and with impulses, *Dynamics of Continuous, Discrete and Impulsive systems-B*, **18**, (2011) 493-520.
- [20] S. Y. Lai and B. Wiwatanapataphe, The asymptotics of global solutions for semi-linear wave equations in two space dimensions, *Dynamics of Continuous, Discrete and Impulsive systems-B*, **18**, (2011) 647-657.

Received February 2012; revised October 2012.

<http://monotone.uwaterloo.ca/~journal/>