

A PREDATORY COMMENSALISM MODEL IN A DETERMINISTIC AND A STOCHASTIC ENVIRONMENT

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Abstract. In the current paper, we propose a two-prey-one-predator system with Holling type II predation functional response where prey species consumes the remains of the other prey species' carcass given by their predator. Mathematical analysis of the proposed model equations with regard to non-negative invariance including stability and bifurcation are carried out. Next, we extend the deterministic system to a stochastic system by incorporating the Gaussian white noise terms in the growth equations of both prey and predator species. We determine fluctuation intensities for the stochastic dynamical system by using Laplace methods. Numerical simulations are exhibited to justify the analytical findings.

Keywords. predator-prey model; commensalism; boundedness; permanence; Hopf bifurcation; global stability; white noise; fluctuation intensity.

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