

EXPONENTIAL STABILITY AND INSTABILITY IN FINITE DELAY NONLINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract. Lyapunov functionals are employed to obtain sufficient conditions that guarantee exponential stability of the zero solution of the nonlinear Volterra Integro-differential equation with uniformly distributed finite delay

$$x'(t) = - \int_{t-r}^t a(t, s)g(x(s))ds,$$

where the functions $a(t, s)$ and $g(x)$ are continuous on their respective domains and $r > 0$. In addition, we will obtain criteria for instability.

Keywords. Nonlinear; Volterra; Exponential stability; Lyapunov functional; Instability.

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