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BOUNDED, L^1, AND ASYMPTOTICALLY STABLE SOLUTIONS OF A PERTURBED NONLINEAR INTEGRAL EQUATION

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Abstract. In this paper we study the existence of a unique continuous bounded solution and the L^1 property of that solution of the perturbed nonlinear Volterra integral equation

$$x(t) = a(t) - \int_0^t C(t,s)[x(s) + g(s, x(s))]ds, t \geq 0. \tag{I}$$

We also study the asymptotic stability of solutions of this equation.

To study qualitative properties of solutions of this equation researchers normally use the equivalent resolvent equation

$$x(t) = a(t) - \int_0^t R(t,s)a(s)ds - \int_0^t R(t,s)g(s, x(s))ds, \tag{R}$$

along with the assumption that the resolvent function R is integrable in some sense. Under certain assumptions on functions a, C, and g we study the existence and the L^1 property of the solution of (I) using the equivalent equation (R) and then we show that one would get exactly the same results under the same assumptions if the equation (I) is used directly. This shows that there is no need to use the equivalent equation (R) to study these properties under the assumptions we considered. For the existence of solution we use the contraction principle, and for the L^1 property we use Liapunov's method.

We study the asymptotic stability of solutions of (I), using the equivalent resolvent equation (R).

Keywords. Integral equation, resolvent, bounded, L^1, asymptotically stable solutions.

AMS (MOS) subject classification: 45D05, 45A05.

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