

SOLUTIONS OF A NONLINEAR FOURTH ORDER PERIODIC BOUNDARY VALUE PROBLEM FOR DIFFERENCE EQUATIONS

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Abstract. The authors consider the fourth order periodic boundary value problem

$$\Delta^4 u(t-2) - \Delta(p(t-1)\Delta u(t-1)) + q(t)u(t) = f(t, u(t)), \quad t \in [1, N]_{\mathbb{Z}},$$

$$\Delta^i u(-1) = \Delta^i u(N-1), \quad i = 0, 1, 2, 3,$$

where $N \geq 1$ is an integer, $p \in C([0, N]_{\mathbb{Z}}, \mathbb{R})$, $q \in C([1, N]_{\mathbb{Z}}, \mathbb{R})$, and $f \in C([1, N]_{\mathbb{Z}} \times \mathbb{R}, \mathbb{R})$. They obtain sufficient conditions for the existence of one and two solutions of the problem. The analysis is based mainly on the variational method and critical point theory.

Keywords. Discrete boundary value problem, fourth order, solutions, variational methods.

AMS (MOS) subject classification: 39A10, 34B15.

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