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ASYMPTOTICALLY PERIODIC SOLUTIONS OF FRACTIONAL DIFFERENTIAL EQUATIONS

T. A. Burton[1] and Bo Zhang[2]

[1]Northwest Research Institute
732 Caroline St., Port Angeles, WA 98362

[2]Department of Mathematics and Computer Science
Fayetteville State University
Fayetteville, NC 28301

Abstract. In three recent papers investigators have shown that a linear fractional differential equation can not have a periodic solution. This raises two fundamental questions: What are the properties of the out-put function if the in-put function is periodic? What are the properties of perturbations that will leave the out-put function unchanged? We answer both questions here. The out-put function is asymptotically periodic and it is unchanged by perturbations which are $L^1[0,\infty)$ and by perturbations which tend to zero as $t \to \infty$ with these perturbations applied simultaneously in the damping and the forcing terms. We also find a limiting equation which this periodic function satisfies. The methods used include limiting equation techniques and fixed point methods involving both contractions and Krasnoselskii-Schaefer type.

Keywords. fixed points, fractional differential equations, integral equations, asymptotically periodic solutions

AMS (MOS) subject classification: 34A08, 47G05, 34D20.

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