

## **BLOW-UP AND GLOBAL EXISTENCE FOR A CLASS OF NONLOCAL WAVE EQUATION**

Rong Zeng<sup>1</sup> and Chunlai Mu<sup>2</sup>

<sup>1</sup>School of Economic Mathematics  
Southwestern University of Finance and Economics, Chengdu, P. R. China

<sup>2</sup>College of Mathematics and Statistics  
Chongqing University, Chongqing, P. R. China

**Abstract.** In this paper, we study the initial-value problem for a general class of nonlinear nonlocal wave equation. Local existence and sufficient conditions for finite time blow-up and as well as global existence of solutions of the problem are established.

**Keywords.** Nonlocal Cauchy problem; Global existence; Blow up.

**AMS (MOS) subject classification:** 74B20

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