

THE GLOBAL SOLUTIONS FOR A GENERALIZED CAMASSA-HOLM EQUATION WITH NONLINEARITY

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Abstract. The Kato theorem for abstract differential equations is used to establish the local well-posedness of the solution for a nonlinear dissipative Camassa-Holm equation in the space $C([0, T], H^s(R)) \cap C^1([0, T], H^{s-1}(R))$ with $s > \frac{3}{2}$. A sufficient condition for the existence of weak solutions of the equation in the lower order Sobolev space $H^s(R)$ with $1 \leq s \leq \frac{3}{2}$ is developed. In addition, it is shown that if the solution belongs to the space $L^1(R)$ and the initial value satisfies certain sign condition, then there exists a unique global strong solution in the space $C([0, \infty), H^s(R)) \cap C^1([0, \infty), H^{s-1}(R))$ with $s > \frac{3}{2}$.

Keywords. Global solution; Dissipative Camassa-Holm equation; High order nonlinear terms

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