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ASYMPTOTIC TO RAREFACTION WAVES AND VACUUM FOR NAVIER-STOKES EQUATIONS WITH DENSITY-DEPENDENT VISCOSITY

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Abstract. This paper deals with the large time behavior of solutions to the compressible Navier-Stokes equations with density-dependent viscosity coefficients. Assume that the Riemann solution to the Euler equations consists of two rarefaction waves separated by vacuum. We show that if the initial end states correspond to those of the Riemann solution, then the solutions for the Navier-Stokes equations are globally defined and converge to the Rarefaction waves and vacuum as time tends to infinity.

Keywords. Navier-Stokes equations, Euler equations, Rarefaction waves, Density-dependent, Vacuum.

AMS (MOS) subject classification: 35Q30,76N15, 35L65.

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