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MATHEMATICAL MODELLING AND CONTROL OF AN AUTONOMOUS VEHICLE WITH REAR-WHEEL-DRIVE

C. Frangos, Pr.Eng.

P. O. Box 2364, Brooklyn Square
Pretoria 0075, South Africa
email: cfrangos123@gmail.com

Abstract. This work proposes a novel application of Lagrangian mechanics and nonlinear control involving an autonomous vehicle. The vehicle has two front wheels and two rear wheels. It is assumed that the mechanical configuration of the vehicle results in a generalized steering torque associated with the steering angle of one of the front wheels, and a generalized driving torque associated with the rotation angle of the drive system. The steering angles of the front wheels are coupled via a nonlinear constraint. Furthermore, the drive system drives the rear wheels via a differential gear-box and side-shafts. Methods from Lagrangian mechanics are applied to derive the kinematic and dynamic models of the vehicle subject to nonholonomic constraints. The nonholonomic constraints arise due to the assumption that all four wheels roll without slipping. A procedure is proposed for designing nonlinear feedback control laws for the generalized driving and steering torques such that a point on the vehicle asymptotically tracks smooth reference paths in the (X, Y)-plane.

Keywords. Lagrangian mechanics, Nonholonomic constraints, Generalized applied forces, Generalized constraint forces, Nonlinear control, Autonomous vehicle, Rear-wheel-drive, Differential gear-box.

AMS (MOS) subject classification: 70E, 70Q05, 93B, 93C.

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