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ASYMPTOTIC BEHAVIOR OF SOLUTIONS OF SECOND-ORDER SUB-LINEAR DELAY DIFFERENTIAL EQUATIONS WITH IMPULSES

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Abstract. This paper is concerned with second-order sub-linear delay differential equations with impulses of the form

$$(r(t)x^{'}(t))^{'} + p(t)x^{'}(t) + q(t)x^{\theta}(t-\delta) = 0,\ t \neq t_k,$$
$$x^{'}(t_k^{+}) = I_k(x^{'}(t_k)),\ x(t_k^{+}) = J_k(x(t_k)),\ t = t_k,\ k = 1, 2, \cdots,\ t \geq t_0.$$

By impulsive differential inequality and Riccati transformation, sufficient conditions of asymptotic behavior of all solutions of second-order nonlinear delay differential equations with impulses are obtained. An example is also inserted to illustrate the impulsive effect.

Keywords. Asymptotic behavior, impulsive differential inequality, Riccati transformation, second-order sub-linear delay differential equation, impulse.

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1 Introduction

The theory of impulsive differential equations is now being recognized to be not only richer than the corresponding theory of differential equations without impulses but also represents a more natural framework for mathematical modelling of many real-world phenomena [1, 2]. It is meaningful to investigate the behavior of all solutions of impulsive differential equations. There are many papers have devoted to the oscillation criteria of second-order differential equations with impulses [3-7], and to asymptotic behavior of all solutions of differential equation without impulses [8]. Recently, in [9], for the problem with impulses

$$(r(t)x^{'}(t))^{'} + p(t)x^{'}(t) + f(t, x(t)) = 0,\ t \neq t_k, \tag{1}$$
$$x^{'}(t_k^{+}) = I_k(x^{'}(t_k)),\quad x(t_k^{+}) = J_k(x(t_k)),\ t = t_k,\ k = 1, 2, \cdots,\ t \geq t_0, \tag{2}$$

sufficient conditions of the asymptotic behavior of solutions of (1), (2) are derived by impulsive differential inequality and Riccati transformation.

Motivated by [9], by impulsive differential inequality and Riccati transformation, we devote to the asymptotic behavior of all solutions of second-order sub-linear delay differential equations with impulses of the form

$$(r(t)x^{'}(t))^{'} + p(t)x^{'}(t) + q(t)x^{\theta}(t-\delta) = 0,\ t \neq t_k, \tag{3}$$