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TWO NEW APPROACHES TO BARBASHIN THEOREM

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Abstract. The paper is devoted to the study of the uniform exponential stability of evolution families. Following the idea of unifying the discrete-time versions of Barbashin theorem and Datko theorem, necessary and sufficient conditions for the uniform exponential stability are given. As particular cases, the discrete variants for the uniform exponential stability of some well-known stability results due to Datko and Barbashin are obtained.

Keywords. Exponential stability, Datko theorem, Barbashin theorem, evolution families, evolution semigroups.

AMS (MOS) subject classification: 34D05, 39A12.

1 Introduction

We denote by $\mathbb{R}_+$ the set of all non-negative real numbers, by $\mathbb{N}$ the set of all non-negative integers and we put $\mathbb{N}_p := \{j \in \mathbb{N} : \ j \geq p\}$. We consider $T := \{(t,s) : t \geq s \geq 0\}$. Throughout the paper, $(\mathbb{X}, \|.\|)$ denotes a Banach space, while $\mathcal{L}(\mathbb{X})$ denotes the Banach algebra of bounded linear operators acting on $\mathbb{X}$. For each operator B, $D(B)$ denotes the domain of B. A family $\Big\{U(t,s)\Big\}_{t\geq\ s\geq\ 0} \subset \mathcal{L}(\mathbb{X})$ is called to be an evolution family if

- the identity on $\mathbb{X}$ can obtained as $U(t,t)$ for every $t \geq 0$,
- the cocycle property $U(t,s) = U(t,r)U(r,s)$ holds for all $t \geq r \geq s \geq 0$,
- the mapping $(t,s) \to U(t,s)x$ is continuous for every $x \in \mathbb{X}$,
- there exist M, ω such that

$$\|U(t+s,s)\| \leq Me^{\omega t}, \text{ for every } (t,s) \in \mathbb{R}_+^2. \tag{1.1}$$

Evolution families that we consider, arise as solution operators of the so-called abstract Cauchy problem. Such a problem states as follows: Let $A(s) : D(A(s)) \subset \mathbb{X} \to \mathbb{X}$ be a (possibly unbounded) linear operator on Banach