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BLOW-UP FOR A PARABOLIC SYSTEM WITH NONLINEAR MEMORY

Keng Deng and Zhihua Dong

Department of Mathematics

University of Louisiana at Lafayette, Lafayette, LA 70504-1010, USA

Abstract. In this paper, we consider nonnegative solutions of

$$\begin{array}{llll} u_t = \Delta u + u^{q_1}\int_0^t v^{p_1}(x,s)ds & v_t = \Delta v & x \in \Omega, & t > 0, \\ \frac{\partial u}{\partial \nu} = 0 & \frac{\partial v}{\partial \nu} = v^{q_2}\int_0^t u^{p_2}(x,s)ds & x \in \partial\Omega, & t > 0, \\ u(x,0) = u_0(x) & v(x,0) = v_0(x) & x \in \overline{\Omega}. & \end{array}$$

We prove that if $0 \le q_1, q_2 \le 1$ and $p_1p_2 \le (1-q_1)(1-q_2)$, all solutions are global; while if $q_1 > 1$ or $q_2 > 1$ or $p_1p_2 > (1-q_1)(1-q_2)$, every solution blows up in finite time. We also show that if $q_1 = q_2 = 0$ and $p_1, p_2 \ge 1$, then blow-up can occur only on the boundary.

Keywords. Parabolic system with memory, global existence, finite time blow-up, blow-up on the boundary.

AMS (MOS) subject classification: 35B44, 35K51, 35K61.

1 Introduction

In this paper, we study blow-up for the following parabolic system with nonlinear memory:

$$\begin{array}{llll} u_t = \Delta u + u^{q_1}\int_0^t v^{p_1}(x,s)ds & v_t = \Delta v & x \in \Omega, & t > 0, \\ \frac{\partial u}{\partial \nu} = 0 & \frac{\partial v}{\partial \nu} = v^{q_2}\int_0^t u^{p_2}(x,s)ds & x \in \partial\Omega, & t > 0, \\ u(x,0) = u_0(x) & v(x,0) = v_0(x) & x \in \overline{\Omega}, & \end{array} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with boundary $\partial\Omega \subset C^{1+\gamma}(0 < \gamma < 1)$, ν is the outward normal, $p_i, q_i \ge 0$ $(i = 1, 2)$, and $u_0(x)$, $v_0(x)$ are nonnegative functions such that

$$\frac{\partial u_0}{\partial \nu} = 0 \qquad \frac{\partial v_0}{\partial \nu} = 0 \quad \text{for } x \in \partial\Omega. \tag{1.2}$$

There are two main sources of motivation for the present study of (1.1). The first one originates with a special case of a model that has been formulated for capillary growth in solid tumors as initiated by angiogenic growth factors [9]:

$$\begin{array}{lll} u_t = \Delta u & x \in \Omega, & t > 0, \\ \frac{\partial u}{\partial \nu} = u^q\int_0^t u^p(x,s)ds & x \in \partial\Omega, & t > 0, \\ u(x,0) = u_0(x) & x \in \overline{\Omega}. & \end{array} \tag{1.3}$$