Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **19 (2012)** 717-728
 http://www.watam.org

A CLASS OF NONLINEAR BVPS FOR FIRST-ORDER IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS

Jing Zhao[1,2] Jiangfeng Han[1] and Yanping Deng[1]

[1]College of Science
Guangxi University for Nationalities, Nanning, Guangxi, 530006, P. R. China

[2]Guangxi Key laboratory of hybrid computation and IC design analysis
E-mail: jingzhao100@126.com

Abstract. This paper is devoted to the existence of extremal solutions for first-order impulsive integro-differential equations. The approach relies on the use of the method of upper and lower solutions in reversed order and monotone iterative technique. By establishing a new comparison result, we show the existence of extremal solutions under certain assumptions.

Keywords. Impulsive integro-differential equation, Upper and lower solutions in reversed order, Monotone iterative technique, Nonlinear boundary value problem.

AMS (MOS) subject classification: 34G25,39J35,45N05.

1 Introduction

Consider the following first-order impulsive integro-differential equations with nonlinear boundary conditions

$$\begin{cases} x'(t) = f(t, x(t), (Tx)(t), (Sx)(t)) & t \in J^- \\ \Delta x(t_k) = I_k(x(t_k)) & k = 1, 2, \dots, m \\ g(x(0), x(T)) = 0 \end{cases} \tag{1.1}$$

where $f \in C(J \times R^3, R)$, $g \in (R^2, R)$, $J = [0, T]$, $J^- = J - \{t_1, t_2, \dots, t_m\}$, $0 = t_0 < t_1 < t_2 < \cdots < t_m < t_{m+1} = T$, $I_k \in C(R, R)$, $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+)$ and $x(t_k^-)$ denote the right and the left limit of $x(t)$ for $t = t_k, (k = 1, 2, \dots, m)$ respectively,

$$(Tx)(t) = \int_0^t k(t,s)x(s)ds, \qquad (Sx)(t) = \int_0^T h(t,s)x(s)ds$$

$k \in C(D, R^+)$, $D = \{(t,s) \in J \times J : \ t \geq s\}$, $h \in C(J \times J, R^+)$, $R^+ = [0, \infty)$.

Recently, the general theory of impulsive differential equations has become an important aspect of differential equations for its extensively application (cf. [1],[5],[8],[10],[15-18]). As an important branch, boundary value problems (BVPS) have drawn much attention in [3],[6-7],[14].