Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **19 (2012)** 675-691
 http://www.watam.org

HOPF BIFURCATION ANALYSIS OF A SPECIAL SEIR EPIDEMIC MODEL WITH NONLINEAR INCIDENCE RATES

Changjin Xu[1] and Maoxin Liao[2]

[1]Guizhou Key Laboratory of Economics System Simulation,
Guizhou University of Finance and, Economics, Guiyang 550004, PR China

[2]School of Mathematics and Physics
University of South China, Hengyang 421001, PR China

Corresponding author email:xcj403@126.com

Abstract. In this paper, a special SEIR epidemic model with nonlinear incidence rates is considered. By analyzing the associated characteristic transcendental equation, its linear stability is investigated and Hopf bifurcation is demonstrated. Some explicit formulae determining the stability and the direction of the Hopf bifurcation periodic solutions bifurcating from Hopf bifurcations are obtained by using the normal form theory and center manifold theory. Some numerical simulation for justifying the theoretical analysis are also provided. Finally, biological explanations and main conclusions are given.

Keywords. SEIR epidemic model, Stability, Hopf bifurcation, Periodic solution.

AMS (MOS) subject classification: 34K20, 34C25.

1 Introduction

In recent years, great attention has been paid to the dynamics properties of (including stable, unstable, persistent and oscillatory behavior) the epidemic models which have significant biological background. Many excellent and interesting results have been obtained [6, 8, 13, 14, 17, 20-22]. It is well known that epidemic models investigate the transmission dynamics of infectious diseases in host population. In this paper, we assume that disease spreads in a single host population through direct contact of hosts and a host stays in a latent period before becoming infectious after the initial infection. An infectious host may die from disease or recover with acquired immunity to the disease at the infectious stage. The host population is partitioned into four classes: the susceptible, exposed (latent), infectious, and recovered with sizes denoted by S, E, I, and R, respectively. The host total population $N = S + E + I + R$. Then, we consider the following differential equations:

$$\begin{cases} \dot{S}(t) = \mu - \mu S - \alpha I^p S^q, \\ \dot{E}(t) = \alpha I^p S^q - (\epsilon + \mu)E, \\ \dot{I}(t) = \epsilon E - (\gamma + \mu)I, \\ \dot{R}(t) = \gamma I - \mu R, \end{cases} \tag{1}$$