

## THE $\epsilon$ -EFFICIENCY CONDITIONS FOR MULTIOBJECTIVE FRACTIONAL PROGRAMMING PROBLEMS

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**Abstract.** A class of parametric sufficient  $\epsilon$ -efficiency conditions and  $\epsilon$ -efficient solutions to multiobjective fractional programming problems based on  $(\rho, \eta, \theta)$ -invexity of higher orders are investigated. The obtained results present a greater degree of generality in nature, while unifying most of the results on the generalized invexities in the literature.

**Keywords.** Multiobjective fractional programming; Higher order invexity;  $\epsilon$ -efficient solutions;  $\epsilon$ -efficiency conditions.

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### 1 Introduction

Consider a multiobjective fractional programming problem (based on the generalized  $(\rho, \eta, \theta)$ -invexity of higher order ( $r \geq 1$ ) for differentiable functions):

(P)

$$\text{Minimize}_{x \in X} \left( \frac{f_1(x)}{g_1(x)}, \dots, \frac{f_p(x)}{g_p(x)} \right)$$

subject to  $x \in X$  such that  $h_j(x) \leq 0$  for  $j = 1, \dots, m$ , where  $f_i, g_i, i \in \{1, 2, \dots, p\}$  are real-valued functions. Here  $X$  is an open convex subset of  $\mathbb{R}^n$  (the  $n$ -dimensional Euclidean space), while  $\eta, \theta : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  are two vector-valued functions.

We explore parametric and semiparametric sufficient conditions for  $\epsilon$ -efficient solvability of (P) based on the generalized  $(\rho, \eta, \theta)$ -invexity of higher order. Let  $Q = \{x \in X : h_j(x) \leq 0, j = 1, \dots, m\}$  denote the feasible set of (P). We observe that (P) is equivalent the parametric multiobjective non-fractional programming problem:

(P $\lambda$ )

$$\text{Minimize}_{x \in X} (f_1(x) - \lambda_1 g_1(x), \dots, f_p(x) - \lambda_p g_p(x)),$$