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OSCILLATION THEOREMS FOR SECOND ORDER SUBLINEAR DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. In this paper, the oscillatory behavior of the second order sublinear dynamic equation

$$(r(t)x^{\Delta}(t))^{\Delta} + p(t)f(x(\sigma(t))) = 0, \tag{1}$$

is studied under the assumption

$$R(t) := \int_T^{\infty} \frac{\Delta t}{r(t)} < \infty,$$

where $r, p \in C(\mathbb{T}, \mathbb{R})$, $r(t) > 0$, $\mathbb{T}$ is a time scale, $T \in \mathbb{T}$, $f : \mathbb{R} \to \mathbb{R}$ is continuously differentiable and satisfies $f'(x) > 0$, $xf(x) > 0$ for $x \neq 0$, and the sublinearity conditions

$$0 < \int_0^{\epsilon} \frac{dx}{f(x)}, \quad \int_{-\epsilon}^0 \frac{dx}{f(x)} < \infty, \text{ for all } \epsilon > 0.$$

When the coefficient function $p(t)$ is allowed to be negative for arbitrarily large values of t, we establish sufficient conditions for oscillation of all solutions of equation (1). In particular, as applications of the main results, we show that the sublinear differential equation

$$(r(t)x'(t))' + p(t)x^{\alpha}(t) = 0, \quad 0 < \alpha < 1$$

is oscillatory, if

$$\int^{\infty} R(t)p(t)\, dt = \infty$$

and the sublinear difference equation

$$\Delta(r(n)\Delta x(n)) + p(n)x^{\alpha}(n+1) = 0, \quad 0 < \alpha < 1$$

is oscillatory, if

$$\sum^{\infty} R(n+1)p(n) = \infty.$$

Keywords. oscillation; sublinear; Emden–Fowler; dynamic equation; time scales

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