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COMPLETE CLASSIFICATION OF SOLUTIONS TO A PARABOLIC EQUATION WITH NONLOCAL BOUNDARY CONDITION

Yuzhu Han[1] and Wenjie Gao[2]

[1]Institute of Mathematics, Jilin University, Changchun, 130012, PR China

[2]Department of Mathematics, Jilin University, Changchun, 130012, PR China

Corresponding author email:hanyuzhu2003@yahoo.cn

Abstract. In this paper, the authors investigate a quasi-linear parabolic equation $u_t = f(u)(\Delta u + a\int_\Omega u(y,t)dy)$ with nonlocal boundary condition $u(x,t) = \int_\Omega k(x,y)u(y,t)dy,\ x \in \partial\Omega$, where Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$. Under some hypotheses imposed on $f(s)$ and $k(x,y)$, they give a complete classification for the solutions to blow-up in finite time or to exist globally. Meanwhile, they obtain an upper bound of the solution to this problem for arbitrary kernel $k(x,y)$, which, to the best of the authors' knowledge, has never been found in the previous works. Blow-up set and precise blow-up rate of the solutions are also investigated in this paper.

Keywords. Nonlocal boundary condition; Nonlocal source; Complete classification; Blow-up profile.

AMS (MOS) subject classification: 35K55, 35K57.

1 Introduction

In this paper, we are interested in the following quasi-linear parabolic equation with nonlocal boundary condition and a nonlocal source

$$\begin{cases} u_t = f(u)(\Delta u + a\int_\Omega u(y,t)dy), & x \in \Omega,\ t > 0, \\ u(x,t) = \int_\Omega k(x,y)u(y,t)dy, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = u_0(x), & x \in \Omega, \end{cases} \tag{1}$$

where a is a positive constant, Ω is a bounded domain in R^N with smooth boundary $\partial\Omega$.

In the past several decades, many physical phenomena have been formulated into nonlocal parabolic equations (see [1, 16, 19, 20]), and in recent years, parabolic equations or systems with nonlocal sources have also been extensively studied. It has been suggested that nonlocal growth terms present more realistic model in physics for compressible reactive gases; see [2]. Equation in (1) arises in the study of the flow of a fluid through a porous medium (when $f(s) = s^p,\ (0 < p < 1)$) with an integral source and in the