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POSITIVE SOLUTION FOR A DISCRETE FRACTIONAL PERIODIC BOUNDARY VALUE PROBLEM

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Abstract. In this work, existence criteria for a positive solution for the following discrete fractional periodic boundary value problem

$$\begin{aligned} \left({}_{\alpha-1}\Delta^{\alpha} u\right)(t) &= \lambda u(t+\alpha-1) + f(t+\alpha-1, u(t+\alpha-1)) \\ u(\alpha-1) &= u(\alpha-1+T), \end{aligned}$$

is established by using a fixed point theorem for operators on cones. An example is also included to illustrate the importance of our results.

Keywords. Discrete fractional calculus, positive solution, periodic boundary value problem.

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1 Introduction

Discrete Fractional Calculus, using Riemann–Liouville operators, was introduced in the work by Miller and Ross in 1989 [18]. Since then, the theory is being developed in various directions [3, 4, 6, 10, 11, 12, 13, 16, 17]. In particular, some real world phenomena are being studied with the help of these discrete fractional operators [5].

On the other hand, some researchers are using the so called Caputo's fractional operator in order to deduce some discrete fractional results [1, 8].

However, in none of the above cited works is a discrete fractional boundary value problem with periodic boundary conditions studied. This turned out to be our motivation to research within this subject. Indeed, we study the discrete fractional periodic boundary value problem (FPBVP)

$$\left({}_{\alpha-1}\Delta^{\alpha} u\right)(t) = \lambda u(t+\alpha-1) + f(t+\alpha-1, u(t+\alpha-1)),$$

subject to

$$u(\alpha-1) = u(\alpha-1+T),$$