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SOLVABILITY OF WEAKLY COUPLED SECOND ORDER SYSTEMS WITH RAPIDLY OSCILLATING NONLINEARITIES

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Abstract. We study the existence of periodic solutions for a nonlinear system of second order ordinary differential equations. Assuming suitable conditions, we prove the existence of at least one solution applying topological degree methods. Instead of a Nirenberg type condition, we shall assume that each coordinate of the nonlinearity satisfies a one-side Landesman-Lazer type condition, but it might present rapid oscillations on the other side.

Keywords. Nonlinear second order systems; periodic solutions; Landesman-Lazer conditions; rapid oscillations; topological degree methods.

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1 Introduction

We study a nonlinear system of second order differential equations for a vector function $u : [0, T] \to \mathbb{R}^N$ satisfying

$$u'' + g(t, u) = 0 \qquad 0 < t < T \tag{1}$$

under periodic boundary conditions:

$$u(0) = u(T), \qquad u'(0) = u'(T). \tag{2}$$

We shall assume that the nonlinearity $g : [0, T] \times \mathbb{R}^N \to \mathbb{R}^N$ is continuous. However, in contrast with the standard Landesman-Lazer conditions for $N = 1$ (see [7], [9]) and their generalization by Nirenberg in [10] for $N > 1$, we shall not assume that g has radial limits at infinity. In fact, g is not necessarily bounded.

For convenience, let us briefly recall the Nirenberg condition for the case $u''(t) + g(u(t)) = p(t)$. When $N = 1$, the Landesman-Lazer theorem establishes the existence of T-periodic solutions of the problem under one of the