Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **19 (2012)** 485-500
 http://www.watam.org

SEVERAL PROPERTIES OF SOLUTIONS TO A NONLINEAR DISPERSIVE WAVE EQUATION

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Abstract. In this paper, a nonlinear dispersive wave equation is investigated. The sufficient conditions about persistence properties of strong solution and infinite propagation speed for the equation are established. Under certain conditions, the existence and uniqueness of the global strong solution is shown to be true.

Keywords. Nonlinear dispersive wave equation; Global existence; Persistence property; Unique continuation; Infinite propagation speed

AMS (MOS) subject classification: 35G25,35L05,35Q35

1 Introduction

In this article, we will consider the nonlinear dispersive wave equation

$$u_t - u_{txx} + (a+b)uu_x - au_xu_{xx} - buu_{xxx} + \lambda(u - u_{xx}) = 0, \tag{1}$$

where $a > 0, b > 0$ and $\lambda \geq 0$ are arbitrary constants, u is the fluid velocity in the x direction, $\lambda(u - u_{xx})$ represents the weakly dissipative term. It should be noted that when $\lambda = 0$, Eq.(1) is different from b-family equation [1], for example, Eq.(1) can be treated as Camassa-Holm type equation if $a = \frac{2}{3}$ and $b = \frac{1}{3}$. We also notice that Eq.(1) with $\lambda = 0$ is a special case of the shallow water equation derived by Constantin and Lannes [4].

For $a = 2$, $b = 1$ and $\lambda = 0$ in Eq.(1), Eq.(1) becomes the famous Camassa-Holm equation derived by Camassa and Holm [1]

$$u_t - u_{txx} + 3uu_x - 2u_xu_{xx} - uu_{xxx} = 0, \quad t > 0, \quad x \in R, \tag{2}$$

where $u(x,t)$ represents the free surface above a flat bottom. As a model to describe the shallow water motion, Eq.(2) has a bi-Hamiltonian structure and infinite conservation laws and is completely integrable. Recently, some significant results of dynamical behaviors have been obtained for the shallow water wave equation. For example, the local well-posedness of solution for initial